Second Order Derivatives

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when:

(a) $y = x^2 + 4x - 3$

(b) $y = 5x^3 + x^2 + 8x - 3$

(c) $y = x^4 - 7x^2$

(d) $y = x^2 - \frac{2}{x}$

(a) $\frac{dy}{dx} = 2x + 4$ $\frac{d^2y}{dx^2} = 2$

(b) $\frac{dy}{dx} = 15x^2 + 2x + 8$

$\frac{d^2y}{dx^2} = 30x + 2$

(c) $\frac{dy}{dx} = 4x^3 - 14x$ $\frac{d^2y}{dx^2} = 12x^2 - 14$

(d) $\frac{dy}{dx} = 2x + \frac{2}{x^2}$ $\frac{d^2y}{dx^2} = 2 - \frac{4}{x^3}$

Find the coordinates of the stationary points on each of these curves. By differentiating for a second time, establish whether these points are maximums or minimums.

(a) $y = 4x^2 - 8x$

(b) $y = 5 + 2x - x^2$

(c) $y = (8 + x)(2 - x)$

(d) $y = x^4 - 8x^2$

(e) $y = 2x^3 - 3x^2 - 12x + 5$

(f) $y = x + \frac{1}{x}$

(a) $(1, -4)$ MINIMUM

(b) $(1, 6)$ MINIMUM

(c) $(-3, 25)$ MAXIMUM

(d) $(0, 0)$ MAXIMUM

$(2, -16)$ MINIMUM

$(-2, -16)$ MINIMUM

(e) $(-1, 12)$ MAXIMUM

$(2, -15)$ MINIMUM

(f) $(1, 2)$ MINIMUM

$(-1, -2)$ MAXIMUM

(a) Find the coordinates of the stationary point on the curve $y = x^3 + 3x^2 + 3x + 1$.

(b) By considering the gradient either side of the stationary point, show that the stationary point is a point of inflection.

(a) $(-1, 0)$ $\frac{dy}{dx} = 3x^2 + 6x + 3$

when $x = -1.1$ $\frac{dy}{dx} = 0.03$

when $x = -0.9$ $\frac{dy}{dx} = 0.03$

Gradient / — / $\therefore$ point of inflection

(a) Find the coordinates of the stationary point on the curve $y = (2 - x)^3$.

(b) By considering the gradient either side of the stationary point, show that the stationary point is a point of inflection.

(b) $(2, 0)$ $\frac{dy}{dx} = -12 + 12x - 3x^2$

when $x = 1.9$ $\frac{dy}{dx} = -0.03$

when $x = 2.1$ $\frac{dy}{dx} = -0.03$

Gradient

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$\therefore$ Point of inflection