

Second Order Derivatives

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when:

- (a) $y = x^2 + 4x - 3$
 (b) $y = 5x^3 + x^2 + 8x - 3$
 (c) $y = x^4 - 7x^2$
 (d) $y = x^2 - \frac{2}{x}$

Find the coordinates of the stationary points on each of these curves. By differentiating for a second time, establish whether these points are maximums or minimums.

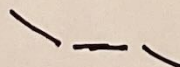
- (a) $y = 4x^2 - 8x$
 (b) $y = 5 + 2x - x^2$
 (c) $y = (8 + x)(2 - x)$
 (d) $y = x^4 - 8x^2$
 (e) $y = 2x^3 - 3x^2 - 12x + 5$
 (f) $y = x + \frac{1}{x}$

(a) Find the coordinates of the stationary point on the curve $y = x^3 + 3x^2 + 3x + 1$.

(b) By considering the gradient either side of the stationary point, show that the stationary point is a point of inflection.

(a) Find the coordinates of the stationary point on the curve $y = (2 - x)^3$.

(b) By considering the gradient either side of the stationary point, show that the stationary point is a point of inflection.

Gradient

 \therefore Point of inflection

$$(a) \frac{dy}{dx} = 2x + 4 \quad \frac{d^2y}{dx^2} = 2$$

$$(b) \frac{dy}{dx} = 15x^2 + 2x + 8$$

$$\frac{d^2y}{dx^2} = 30x + 2$$

$$(c) \frac{dy}{dx} = 4x^3 - 14x \quad \frac{d^2y}{dx^2} = 12x^2 - 14$$

$$(d) \frac{dy}{dx} = 2x + \frac{2}{x^2} \quad \frac{d^2y}{dx^2} = 2 - \frac{4}{x^3}$$

(a) $(1, -4)$ MINIMUM

(b) $(1, 6)$ MINIMUM

(c) $(-3, 25)$ MAXIMUM

(d) $(0, 0)$ MAXIMUM
 $(2, -16)$ MINIMUM
 $(-2, -16)$ MINIMUM

(e) $(-1, 12)$ MAXIMUM
 $(2, -15)$ MINIMUM

(f) $(1, 2)$ MINIMUM
 $(-1, -2)$ MAXIMUM

(a) $(-1, 0) \quad \frac{dy}{dx} = 3x^2 + 6x + 3$

when $x = -1.1 \quad \frac{dy}{dx} = 0.03$

when $x = -0.9 \quad \frac{dy}{dx} = 0.03$

Gradient \nearrow \searrow \therefore point of inflection

(b) $(2, 0) \quad \frac{dy}{dx} = -12 + 12x - 3x^2$

when $x = 1.9 \quad \frac{dy}{dx} = -0.03$

when $x = 2.1 \quad \frac{dy}{dx} = -0.03$