Investigating Powers of Complex Numbers

Let z be the complex number $z = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$ or $z = 2e^{\frac{\pi i}{5}}$

Find the different powers of z in modulus-argument and exponential forms.

Power of z	**Calculation**	**Modulus-Argument Form**	**Exponential Form**
z		$2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$	$z = 2e^{\frac{\pi i}{5}}$
z^2	$2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right) \times 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$	$4\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$	$z = 4e^{\frac{2\pi i}{5}}$
z^3	$4\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right) \times 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$	$8\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$	$z = 8e^{\frac{3\pi i}{5}}$
z^4	$8\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right) \times 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$	$16\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$	$z = 16e^{\frac{4\pi i}{5}}$
z^5	$16\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right) \times 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$	$32(\cos\pi + i\sin\pi)$	$z = 32e^{\pi i}$
z^n		$2^n\left(\cos\frac{n\pi}{5} + i\sin\frac{n\pi}{5}\right)$	$z = 2^n e^{\frac{n\pi i}{5}}$

Now, consider the general complex number $z = r(\cos\theta + i\sin\theta)$ or $z = re^{i\theta}$

Find the different powers of z in modulus-argument and exponential forms.

Power of z	**Calculation**	**Modulus-Argument Form**	**Exponential Form**
z		$r(\cos\theta + i\sin\theta)$	$re^{i\theta}$
z^2	$r(\cos\theta + i\sin\theta) \times r(\cos\theta + i\sin\theta)$	$r^2(\cos 2\theta + i\sin 2\theta)$	$r^2e^{2i\theta}$
z^3	$r^2(\cos 2\theta + i\sin 2\theta) \times r(\cos\theta + i\sin\theta)$	$r^3(\cos 3\theta + i\sin 3\theta)$	$r^3e^{3i\theta}$
z^4	$r^3(\cos 3\theta + i\sin 3\theta) \times r(\cos\theta + i\sin\theta)$	$r^4(\cos 4\theta + i\sin 4\theta)$	$r^4e^{4i\theta}$
z^5	$r^4(\cos 4\theta + i\sin 4\theta) \times r(\cos\theta + i\sin\theta)$	$r^5(\cos 5\theta + i\sin 5\theta)$	$r^5e^{5i\theta}$
z^n		$r^n(\cos n\theta + i\sin n\theta)$	$r^ne^{in\theta}$

We can conclude that:

$$\big(r(\cos\theta + i\sin\theta)\big)^n = r^n(\cos n\theta + i\sin n\theta)$$