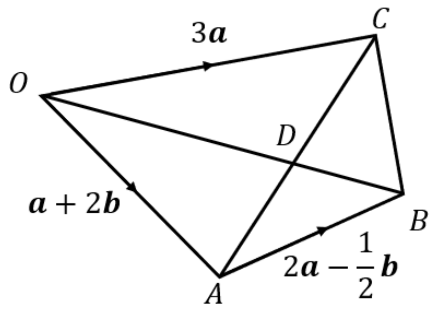
Vector Proof – Equating Coefficients

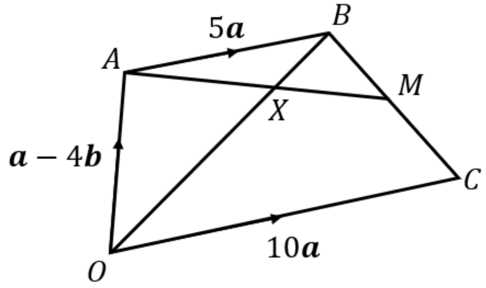
(a)

$OABC$ is a quadrilateral, where $\overrightarrow{OC} = 3\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} + 2\boldsymbol{b}$ and $\overrightarrow{AB} = 2\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$. The point D is on OB and AC such that $OD : OB = \lambda : 1$ and $AD : AC = \mu : 1$. By finding two ways to express the vector $\overrightarrow{OD}$, find the values of λ and μ.



(b)

$OABC$ is a trapezium, where $\overrightarrow{OC} = 10\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} - 4\boldsymbol{b}$ and $\overrightarrow{AB} = 5\boldsymbol{a}$. M is the midpoint of the line BC. The point X is on OB and AM such that $OX : OB = \lambda : 1$ and $AX : AM = \mu : 1$. Find the values of λ and μ and the vector $\overrightarrow{OX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



(c)

In the triangle OAB, $\overrightarrow{OB} = 5\boldsymbol{b}$ and $\overrightarrow{OM} = 2\boldsymbol{a} + 2\boldsymbol{b}$, where M is the midpoint of OA. OC is the line OB produced and $\overrightarrow{OB} = \overrightarrow{BC}$. The point X is on the line AB such that $AX : AB = \lambda : 1$. Given that MXC is a straight line, find the value of λ and the vector $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.