

Fill in the Blanks

Harder Inverse Functions

$f(x)$	Write as $y = \dots$	Swap x and y	Make y the subject				Write as $f^{-1}(x) = \dots$
$f(x) = \frac{x}{x+2}$	$y = \frac{x}{x+2}$	$x = \frac{y}{y+2}$	$xy + 2x = y$	$2x = y - xy$	$2x = y(1 - x)$	$y = \frac{2x}{1 - x}$	$f^{-1}(x) = \frac{2x}{1 - x}$
$f(x) = \frac{x-3}{x}$	$y = \frac{x-3}{x}$	$x = \frac{y-3}{y}$	$xy = y - 3$	$3 = y - xy$	$3 = y(1 - x)$	$y = \frac{3}{1 - x}$	$f^{-1}(x) = \frac{3}{1 - x}$
$f(x) = \frac{x}{2x+1}$	$y = \frac{x}{2x+1}$	$x = \frac{y}{2y+1}$	$2xy + x = y$	$x = y - 2xy$	$x = y(1 - 2x)$	$y = \frac{x}{1 - 2x}$	$f^{-1}(x) = \frac{x}{1 - 2x}$
$f(x) = \frac{3x}{x-5}$	$y = \frac{3x}{x-5}$	$x = \frac{3y}{y-5}$	$xy - 5x = 3y$	$xy - 3y = 5x$	$y(x - 3) = 5x$	$y = \frac{5x}{x - 3}$	$f^{-1}(x) = \frac{5x}{x - 3}$
$f(x) = \frac{x+2}{x-1}$	$y = \frac{x+2}{x-1}$	$x = \frac{y+2}{y-1}$	$xy - x = y + 2$	$xy - y = x + 2$	$y(x - 1) = x + 2$	$y = \frac{x + 2}{x - 1}$	$f^{-1}(x) = \frac{x + 2}{x - 1}$
$f(x) = \frac{x+3}{x+1}$	$y = \frac{x+3}{x+1}$	$x = \frac{y+3}{y+1}$	$xy + x = y + 3$	$xy - y = 3 - x$	$y(x - 1) = 3 - x$	$y = \frac{3 - x}{x - 1}$	$f^{-1}(x) = \frac{3 - x}{x - 1}$
$f(x) = \frac{2x+1}{3-x}$	$y = \frac{2x+1}{3-x}$	$x = \frac{2y+1}{3-y}$	$3x - xy = 2y + 1$	$3x - 1 = 2y + xy$	$3x - 1 = y(2 + x)$	$y = \frac{3x - 1}{2 + x}$	$f^{-1}(x) = \frac{3x - 1}{2 + x}$
$f(x) = \frac{5-2x}{x-3}$	$y = \frac{5-2x}{x-3}$	$x = \frac{5-2y}{y-3}$	$xy - 3x = 5 - 2y$	$xy + 2y = 3x + 5$	$y(x + 2) = 3x + 5$	$y = \frac{3x + 5}{x + 2}$	$f^{-1}(x) = \frac{3x + 5}{x - 3}$