**Algebraic Proof with Multiples**

(a) Show that $4(n+3)-n$ is a multiple of $3$ for all integer values of $n$

(b) Show that $(n+2)^2+3n^2$ is a multiple of $4$ for all integer values of $n$

(c) Show that $(3n-1)^2-(2n+1)^2$ is a multiple of $5$ for all integer values of $n$

(d) Show that

$$(2n+1)(4n-3)-(n+2)^2-n$$

is a multiple of $7$ for all integer values of $n$

(a) $4n+12-n=3n+12$
$=3(n+4)$

(b) $n^2+4n+4+3n^2$
$=4n^2+4n+4=4(n^2+n+1)$

(c) $9n^2-6n+1-4n^2-4n-1$
$=5n^2-10n=5(n^2-2n)$

(d) $8n^2+4n-6n-3-n^2-4n-4-n$
$=7n^2-7n-7=7(n^2-n-1)$

(a) Show that the sum of three consecutive integers is always a multiple of 3

(b) Show that the sum of three consecutive even numbers is always a multiple of 6

(c) Show that the product of two consecutive even numbers is always a multiple of 4

(a) $n+(n+1)+(n+2)=3n+3$
$=3(n+1)$

(b) $2n+(2n+2)+(2n+4)$
$=6n+6=6(n+1)$

(c) $2n(2n+2)=4n^2+4n$
$=4(n^2+n)$

(a) Prove algebraically that the sum of three consecutive square numbers is never a multiple of 3

(b) Prove algebraically that the sum of the squares of any two odd numbers is never a multiple of 4

(c) Prove algebraically that the product of two consecutive odd numbers is never a multiple of 4

(a) $n^2+(n+1)^2+(n+2)^2$
$=n^2+n^2+2n+1+n^2+4n+4$
$=3n^2+6n+5=3(n^2+2n+2)-1$

(b) $(2n+1)^2+(2m+1)^2$
$=4n^2+4n+1+4m^2+4m+1$
$=4(n^2+m^2+n+m)+2$

(c) $(2n+1)(2n+3)$
$=4n^2+8n+3$
$=4(n^2+2n+1)-1$

(a) Prove algebraically that the product of three consecutive even numbers is always a multiple of 8

(b) Prove algebraically that the sum of the cubes of two consecutive even numbers is always a multiple of 8

(c) Prove algebraically that the product of the squares of two odd numbers is always one more than a multiple of 4

(a) $2n(2n+2)(2n+4)$
$=2n\times 2(n+1)\times 2(n+2)$
$=8n(n+1)(n+2)$

(b) $(2n)^3+(2n+2)^3$
$=8n^3+2^3(n+1)^3$
$=8(n^3+(n+1)^3)$

(c) $(2n+1)^2\times(2m+1)^2$
$=(4n^2+4n+1)(4m^2+4m+1)$
$=16n^2m^2+16n^2m+4n^2+16nm^2+16nm+4n+4m^2+4m+1$
$=4(4n^2m^2+4n^2m+4nm^2+4nm+n^2+m^2+n+m)+1$