

Trigonometric Proof

(a) Show that

$$\cos^2\theta - \sin^2\theta \equiv 1 - 2\sin^2\theta$$

(b) Show that $\tan^2\theta \equiv \frac{\sin^2\theta}{1-\sin^2\theta}$

(a) [Using $\sin^2\theta + \cos^2\theta = 1$]

$$\begin{aligned} \cos^2\theta - \sin^2\theta &= (1 - \sin^2\theta) - \sin^2\theta \\ &= 1 - 2\sin^2\theta \end{aligned}$$

(b) [Using $\sin\theta / \cos\theta = \tan\theta$]

$$\begin{aligned} \tan^2\theta &= \frac{\sin^2\theta}{\cos^2\theta} \\ &= \frac{\sin^2\theta}{1 - \sin^2\theta} \quad \left[\begin{array}{l} \text{Using } \sin^2\theta \\ + \cos^2\theta = 1 \end{array} \right] \end{aligned}$$

(a) Prove that

$$\frac{1}{\cos x} - \cos x \equiv \sin x \tan x$$

(b) Show that $1 + \tan^2\theta \equiv \frac{1}{\cos^2\theta}$

(a) $\frac{1}{\cos x} - \cos x$

$$\begin{aligned} &= \frac{1 - \cos^2 x}{\cos x} \quad \left[\text{Using } \sin^2 x + \cos^2 x = 1 \right] \\ &= \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \times \sin x \\ &= \tan x \sin x \quad \left[\text{Using } \tan x = \frac{\sin x}{\cos x} \right] \end{aligned}$$

(a) Prove that

$$\frac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1} \equiv -1$$

(b) Prove that

$$(2\sin x - \cos x)^2 + (\sin x + 2\cos x)^2 \equiv 5$$

(b) $1 + \tan^2\theta$

$$\begin{aligned} &= 1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} \quad \left[\text{Using } \uparrow \right] \\ &= \frac{1}{\cos^2\theta} \quad \left[\text{Using } \sin^2\theta + \cos^2\theta = 1 \right] \end{aligned}$$

(a) Show that

$$\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$$

(b) Show that

$$\frac{1}{\cos^2 x} - 1 \equiv \tan^2 x$$

(a) $\frac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1}$

$$\begin{aligned} &= \frac{\sin^2 x (\sin^2 x + \cos^2 x)}{-(1 - \cos^2 x)} \\ &= \frac{\sin^2 x}{-\sin^2 x} \\ &= -1 \end{aligned}$$

(b) $(2\sin x - \cos x)^2 + (\sin x + 2\cos x)^2$

$$\begin{aligned} &= 4\sin^2 x - 4\sin x \cos x + \cos^2 x \\ &\quad + \sin^2 x + 4\sin x \cos x + 4\cos^2 x \\ &= 5\sin^2 x + 5\cos^2 x \\ &= 5(\sin^2 x + \cos^2 x) \\ &= 5 \end{aligned}$$

$$(a) \frac{\tan x + 1}{\tan x}$$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\sin x \cos x}$$

$$(b) \frac{1}{\cos^2 x} - 1$$

$$= \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$$

$$= \frac{1 - \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x}$$

$$= \tan^2 x$$