Trigonometric Proof

(a) Show that

$$\cos^2\theta - \sin^2\theta \equiv 1 - 2\sin^2\theta$$

(b) Show that $\tan^2\theta \equiv \frac{\sin^2\theta}{1-\sin^2\theta}$

(a) [using $\sin^2\theta+\cos^2\theta=1$]

$\cos^2\theta - \sin^2\theta$

$= (1-\sin^2\theta) - \sin^2\theta$.

$= 1 - 2\sin^2\theta$.

(b) [using $\sin\theta / \cos\theta = \tan\theta$]

$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta}{1-\sin^2\theta}$ [using $\sin^2\theta + \cos^2\theta = 1$]

(a) Prove that

$$\frac{1}{\cos x} - \cos x \equiv \sin x \tan x$$

(b) Show that $1 + \tan^2\theta \equiv \frac{1}{\cos^2\theta}$

(a) $\frac{1}{\cos x} - \cos x$

$= \frac{1-\cos^2 x}{\cos x}$ [using $\sin^2 x + \cos^2 x = 1$]

$= \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \times \sin x$

$= \tan x \sin x$. [using $\tan x = \frac{\sin x}{\cos x}$]

(a) Prove that

$$\frac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1} \equiv -1$$

(b) Prove that

$$(2\sin x - \cos x)^2 + (\sin x + 2\cos x)^2 \equiv 5$$

(b) $1 + \tan^2\theta$

$= 1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$. [using ↑]

$= \frac{1}{\cos^2\theta}$ [using $\sin^2\theta + \cos^2\theta = 1$]

(a) Show that

$$\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$$

(b) Show that

$$\frac{1}{\cos^2 x} - 1 \equiv \tan^2 x$$

(a) $\frac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1}$

$= \frac{\sin^2 x (\sin^2 x + \cos^2 x)}{-(1-\cos^2 x)}$

$= \frac{\sin^2 x}{-\sin^2 x}$

$= -1$

(b) $(2\sin x - \cos x)^2 + (\sin x + 2\cos x)^2$

$= 4\sin^2 x - 4\sin x \cos x + \cos^2 x + \sin^2 x + 4\sin x \cos x + 4\cos^2 x$

$= 5\sin^2 x + 5\cos^2 x$

$= 5(\sin^2 x + \cos^2 x)$

$= 1$.

(a) $\tan x + \frac{1}{\tan x}$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$= \frac{1}{\sin x \cos x}$.

(b) $\frac{1}{\cos^2 x} - 1$

$= \frac{1}{\cos^3 x} - \frac{\cos^2 x}{\cos^2 x}$

$= \frac{1 - \cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x}{\cos^2 x}$

$= \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x}$

$= \tan^2 x$.