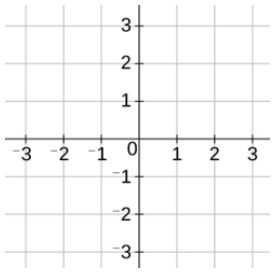
Rotations Using Matrices

(a)

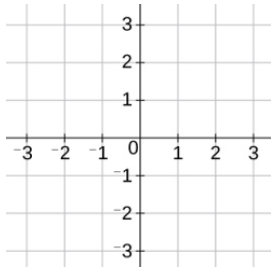
By considering the unit square, determine the matrix which describes a rotation $90°$ clockwise about the origin.



$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(b)

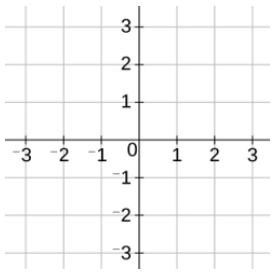
Describe fully the single transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



Rotation $90°$
anti − clockwise about origin

(c)

By considering the unit square, determine the matrix which describes a rotation $180°$ about the origin.



$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(d)

The point $(1, -6)$ is mapped onto the point (a, b) when rotated $90°$ anti-clockwise about the origin. Using matrix algebra, find the values of a and b.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -6 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$a = 6, b = 1$$

(e)

The point (c, d) is mapped onto the point $(2, 4)$ when rotated $270°$ anti-clockwise about the origin. Using matrix algebra, find the values of c and d.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$c = -4, d = 2$$

(f)

A triangle with vertices at $(1, 1)$, $(5, 2)$ and $(4, -1)$ is rotated $180°$ about the origin. Use matrix algebra to find the coordinates of the vertices of the rotated triangle.

Vertices $(-1, -1)$, $(-5, -2)$ *and* $(4, -1)$

(g)

Use matrix algebra to show that a rotation of $90°$ clockwise about the origin, followed by a rotation of $180°$ is equivalent to a rotation of $90°$ anti-clockwise about the origin.

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(h)

The point $(a, 6)$ is mapped onto the point $(b, -4)$ following a rotation of $90°$ anti-clockwise about the origin. Use matrix algebra to find the values of a and b.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ 6 \end{pmatrix} = \begin{pmatrix} b \\ -4 \end{pmatrix}$$

$$a = -4, b = -6$$

(i)

The point $(x, 2y + 6)$ is mapped onto the point $(2x, y - 7)$ following a rotation of $90°$ clockwise about $(0, 0)$. Use matrix algebra to find the values of x and y.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ 2y + 6 \end{pmatrix} = \begin{pmatrix} 2x \\ y - 7 \end{pmatrix}$$

$$x = 5, y = 2$$