

Algebraic Proof with Odds and Evens

Write down algebraic expressions for:

- (a) two different odd numbers
- (b) two consecutive numbers
- (c) two consecutive even numbers
- (d) two different odd numbers squared

- (a) Prove algebraically that the sum of any two odd numbers is always even.
- (b) Prove algebraically that the product of an odd number and an even number is always ~~odd~~ even
- (c) Prove algebraically that the sum of two consecutive numbers is always odd
- (d) Prove algebraically that the product of two consecutive even numbers is always even

- (a) Prove algebraically that the mean of two consecutive odd numbers is always even
- (b) Prove algebraically that the difference between an odd number and an even number is always odd
- (c) Prove algebraically that the mean of three consecutive odd numbers is always equal to the middle number

- (a) Prove algebraically that the sum of the squares of two even numbers is always a multiple of 4
- (b) Prove algebraically that the difference between the squares of two odd numbers is always a multiple of 4
- (c) Prove algebraically that the sum of the squares of two consecutive numbers is always odd

- (a) Show for all integers values of n that $(n+3)^2 - n(n-6) + 2$ is always odd
- (b) Show for all integer values of n that $(3n+5)^2 + (3-n)^2$ is always even

For example...

- (a) $2n+1$ and $2m+1$
- (b) n and $n+1$
- (c) $2n$ and $2n+2$
- (d) $(2n+1)^2$ and $(2m+1)^2$

$$(a) 2n+1+2m+1 = 2n+2m+2 = 2(n+m+1)$$

$$(b) 2n(2m+1) = 4nm+2n = 2(2nm+n)$$

$$(c) n+(n+1) = 2n+1 = 2(n) + 1$$

$$(d) (2n)(2n+2) = 4n^2+2n = 2(2n^2+n)$$

$$(a) \frac{(2n+1)+(2n+3)}{2} = \frac{4n+4}{2} = 2n+2 = 2(n+1)$$

$$(b) (2n+1) - 2m = 2n - 2m + 1 = 2(n-m) + 1$$

$$(c) \frac{(2n+1)+(2n+3)+(2n+5)}{3} = \frac{6n+9}{3} = 2n+3$$

$$(a) (2n)^2 + (2m)^2 = 4n^2 + 4m^2 = 4(n^2 + m^2)$$

$$(b) (2m+1)^2 - (2n+1)^2 = 4m^2 + 4m + 1 - 4n^2 - 4n - 1 = 4(m^2 + m - n^2 - n)$$

$$(c) n^2 + (n+1)^2 = n^2 + n^2 + 2n + 1 = 2(n^2 + n) + 1$$

$$(a) n^2 + 6n + 9 - n^2 + 6n + 2 = 12n + 11 = 2(6n + 6) - 1$$

$$(b) 9n^2 + 30n + 25 + 9 - 6n + n^2 = 10n^2 + 24n + 34 = 2(5n^2 + 12n + 17)$$