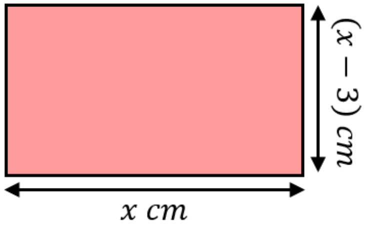
Solving Quadratic Inequalities in Context

(a)

A rectangle has sides of length $x\ cm$ and width $(x-3)\ cm$, as shown. If the area of the rectangle is greater than $10\ cm^2$:



(i) Show that $x^2 - 3x - 10 > 0$

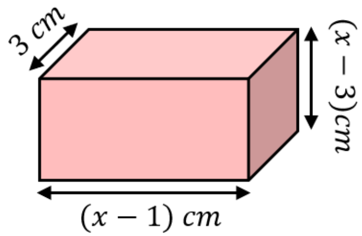
$$x(x-3) > 10$$
$$x^2 - 3x > 10$$
$$x^2 - 3x - 10 > 0$$

(ii) Find the range of possible values of x.

$$(x-5)(x+2) > 0$$
$$x > 5, x < -2$$
$$but\ x > 0, so\ x > 5$$

(b)

A cuboid has dimensions of $3\ cm$, $(x-1)\ cm$ and $(x-3)\ cm$, as shown. If the volume of the cuboid is greater than $45\ cm^3$:



(i) Show that $x^2 - 4x - 12 > 0$

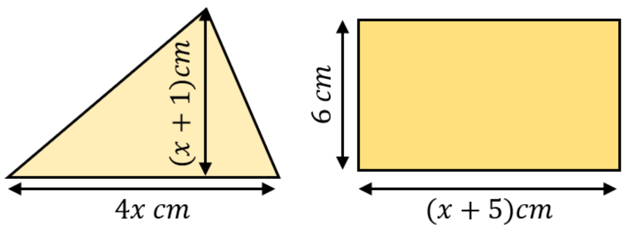
$$3(x-1)(x-3) > 45$$
$$x^2 - 4x + 3 > 15$$
$$x^2 - 4x - 12 > 0$$

(ii) Find the range of possible values of x.

$$(x-6)(x+2) > 0$$
$$x > 6, x < -2$$
$$but\ x > 0, so\ x > 6$$

(c)

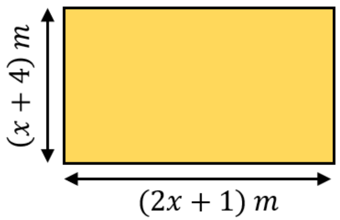
Given that the area of the rectangle is greater than the area of the triangle, find the range of possible values of x.



$$6(x+5) > 0.5 \times 4x(x+1)$$
$$6x + 30 > 2x^2 + 2x$$
$$x^2 - 2x - 15 < 0$$
$$(x-5)(x+3) < 0$$
$$-3 < x < 5, but\ x > 0\ so\ 0 < x < 5$$

(d)

A rectangular lawn has a length of $(2x+1)\ m$ and a width of $(x+4)\ m$, as shown. Given that the area of the lawn is less than $49\ m^2$, find the range of possible values of x.



$$(2x+1)(x+4) < 49$$
$$2x^2 + 9x - 45 < 0$$
$$(2x+15)(x-3) < 0$$
$$-7.5 < x < 3$$
$$but\ x > -\frac{1}{2}, so\ -\frac{1}{2} < x < 3$$