Number and Algebra Proof Revision

(a)

Show that $8\frac{1}{2} - 3\frac{2}{3} = 4\frac{5}{6}$

$$\frac{17}{2} - \frac{11}{3} = \frac{51}{6} - \frac{22}{6}$$
$$= \frac{29}{6} = 4\frac{5}{6}$$

(b)

Show that $\sqrt{80}$ can be written in the form $k\sqrt{5}$ where k is an integer to be found

$$\sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$$
$$k = 5$$

(c)

Show that $0.\dot{7}\dot{5} = \frac{25}{33}$

$$x = 0.\dot{7}\dot{5}$$
$$100x = 75.\dot{7}\dot{5}$$
$$99x = 75$$
$$x = \frac{75}{99} = \frac{25}{33}$$

(d)

Show that $3\frac{5}{8} \div 1\frac{5}{6} = 1\frac{43}{44}$

$$\frac{29}{8} \div \frac{11}{6} = \frac{29}{8} \times \frac{6}{11} = \frac{174}{88}$$
$$= \frac{87}{44} = 1\frac{43}{44}$$

(e)

Show that $0.3\dot{1} = \frac{14}{45}$

$$x = 0.3\dot{1}$$
$$10x = 3.\dot{1}$$
$$100x = 31.\dot{1}$$
$$90x = 28$$
$$x = \frac{28}{90} = \frac{14}{45}$$

(f)

Show that $0.4\dot{2}\dot{7} = \frac{47}{110}$

$$x = 0.4\dot{2}\dot{7}$$
$$10x = 4.\dot{2}\dot{7}$$
$$1000x = 427.\dot{2}\dot{7}$$
$$990x = 423$$
$$x = \frac{423}{990} = \frac{47}{110}$$

(g)

Show that
$$\left(7 - 5\sqrt{3}\right)^2 = a + b\sqrt{3}$$
where a and b are integers to be found

$$(7 - 5\sqrt{3})(7 - 5\sqrt{3})$$
$$= 49 - 35\sqrt{3} - 35\sqrt{3} + 25\sqrt{9}$$
$$= 124 - 70\sqrt{3}$$
$$a = 124, b = -70$$

(h)

Show that the product of an even number and an odd number is always even.

Let odd number = $2n + 1$
Let even number = $2m$
$$2m(2n + 1) = 4mn + 2m$$
$$= 2(2mn + m)$$
which is a multiple of 2 and hence even

(i)

Show that the sum of three consecutive odd numbers is always a multiple of 3

Let consecutive odd numbers be
$2n + 1, 2n + 3$ and $2n + 5$

$$2n + 1 + 2n + 3 + 2n + 5 = 6n + 9$$
$$= 3(2n + 3)$$

which is a multiple of 3

(j)

Show that $\frac{3\sqrt{12}}{2-\sqrt{3}}$ can be written in the form $c + d\sqrt{3}$, where c and d are integers to be found.

$$\frac{3\sqrt{12}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{6\sqrt{12} + 3\sqrt{36}}{4 - 3}$$
$$= \frac{6 \times 2\sqrt{3} + 3 \times 6}{1} = 12\sqrt{3} + 18$$
$$c = 18, d = 12$$

(k)

Show that
$$(3n + 4)(n - 3) + n(n - 3)$$
is a multiple of 4 for all integer values of n

$$(3n + 4)(n - 3) + n(n - 3)$$
$$3n^2 + 4n - 9n - 12 + n^2 - 3n$$
$$= 4n^2 - 8n - 12$$
$$= 4(n^2 - 2n - 3)$$

which is a multiple of 4