Exponentials and Logarithms Revision

(a)

Given that $a = \log_5 8$, write in simplest form in terms of a:

(i) $\log_5 64$ $\quad 2a$

(ii) $\log_5\left(\frac{2\sqrt{2}}{125}\right)$ $\quad \frac{1}{2}a - 3$

(b)

Solve

$$2\log_4(x+1) = \frac{1}{2} + \log_4(3x-1)$$

$x = 3$ or $x = 1$

(c)

Solve

$$2^x \times e^{x+4} = 8$$

giving your answer in terms of natural logarithms.

$$x = \frac{3\ln 2 - 4}{1 + \ln 2}$$

(d)

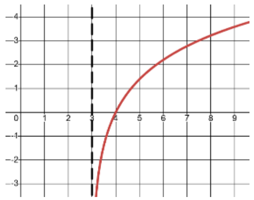
Solve

$$e^x + 12e^{-x} = 8$$

$x = \ln 2$ or $x = \ln 6$

(e)

Sketch the graph of $y = 2\ln(x-3)$, stating any coordinates of intersection with the axes and the equations of any asymptotes.



$(4, 0)$

$x = 3$

(f)

Find the exact coordinates of the point on the curve $y = 4e^{2x} - 3$ where the gradient is 10.

$$\left(\frac{1}{2}\ln\left(\frac{5}{4}\right), 2\right)$$

(g)

A population P of rabbits at time t in months can be modelled by the equation $P = ab^t$ where a and b are constants. The initial population of rabbits is 225 and after 8 months it is 331.

(i) Find the value of a and the value of b to 4 significant figures.

(ii) Use the model to predict the population of rabbits after 1 year.

(iii) Interpret the value of b with reference to the model.

(i) $a = 225, b = 1.049$

(ii) 399 rabbits

(iii) b is the ratio of the number of rabbits from one month to the next

(h)

The mass m grams of a radioactive isotope at time t can be modelled by the equation $m = ae^{-kt}$ where a and k are constants.

(i) Show algebraically that the graph of $\ln m$ against t is linear

(ii) When the graph of $\ln m$ against t is plotted, the gradient of the line is -0.03 and the point $(0, 9)$ lies on the line, find the exact values of a and k.

(i) $\ln m = \ln a + \ln e^{-kt}$

$\ln m = -kt + \ln a$

which is of the form $y = mx + c$, where $-k$ is the gradient and $\ln a$ is the y-intercept.

(ii) $a = e^9$ and $k = 0.03$