Binomial Expansion

(a) Expand and simplify $(x+2)^3$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	x^3	2^0	x^3
3	x^2	2^1	$6x^2$
3	x^1	2^2	$\mathbf{12x}$
1	x^0	2^3	$\mathbf{8}$

$$= x^3 + 6x^2 + \mathbf{12x} + \mathbf{8}$$

(b) Expand and simplify $(x-5)^3$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	x^3	$(-5)^0$	x^3
3	x^2	$(-5)^1$	$-15x^2$
3	$\mathbf{x^1}$	$\mathbf{(-5)^2}$	$\mathbf{+75x}$
1	$\mathbf{x^0}$	$\mathbf{(-5)^3}$	$\mathbf{-125}$

$$= x^3 - 15x^2 + \mathbf{75x} - \mathbf{125}$$

(c) Expand and simplify $(x+y)^4$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	x^4	y^0	$\mathbf{x^4}$
4	x^3	y^1	$\mathbf{4x^3y}$
6	x^2	y^2	$\mathbf{6x^2y^2}$
4	$\mathbf{x^1}$	$\mathbf{y^3}$	$\mathbf{4xy^3}$
1	$\mathbf{x^0}$	$\mathbf{y^4}$	$\mathbf{y^4}$

$$= \mathbf{x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4}$$

(d) Expand and simplify $(2x+1)^4$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	$(2x)^4$	1^0	$16x^4$
4	$\mathbf{(2x)^3}$	$\mathbf{1^1}$	$\mathbf{32x^3}$
6	$\mathbf{(2x)^2}$	$\mathbf{1^2}$	$\mathbf{24x^2}$
4	$\mathbf{(2x)^1}$	$\mathbf{1^3}$	$\mathbf{8x}$
1	$\mathbf{(2x)^0}$	$\mathbf{1^4}$	$\mathbf{1}$

$$= \mathbf{16x^4 + 32x^3 + 24x^2 + 8x + 1}$$

(e) Expand and simplify $(3x-2)^5$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	$(3x)^5$	$(-2)^0$	$\mathbf{243x^5}$
5	$(3x)^4$	$(-2)^1$	$\mathbf{-810x^4}$
10	$\mathbf{(3x)^3}$	$\mathbf{(-2)^2}$	$\mathbf{1080x^3}$
10	$\mathbf{(3x)^2}$	$\mathbf{(-2)^3}$	$\mathbf{-720x^2}$
5	$\mathbf{(3x)^1}$	$\mathbf{(-2)^4}$	$\mathbf{+240x}$
1	$\mathbf{(3x)^0}$	$\mathbf{(-2)^5}$	$\mathbf{-32}$

$$= \mathbf{243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32}$$

(f) Expand and simplify $(4-y)^5$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	4^5	$(-y)^0$	$\mathbf{1024}$
5	$\mathbf{4^4}$	$\mathbf{(-y)^1}$	$\mathbf{-1280y}$
10	$\mathbf{4^3}$	$\mathbf{(-y)^2}$	$\mathbf{+640y^2}$
10	$\mathbf{4^2}$	$\mathbf{(-y)^3}$	$\mathbf{-160y^3}$
5	$\mathbf{4^1}$	$\mathbf{(-y)^4}$	$\mathbf{+20y^4}$
1	$\mathbf{4^0}$	$\mathbf{(-y)^5}$	$\mathbf{-y^5}$

$$= \mathbf{1024 - 1280y + 640y^2 - 160y^3 + 20y^4 - y^5}$$