More Binomial Expansion

(a) Expand and simplify $(x-2)^6$

$$1\times x^6\times(-2)^0$$
$$+6\times x^5\times(-2)^1$$
$$+15\times x^4\times(-2)^2$$
$$+20\times x^3\times(-2)^3$$
$$+15\times x^2\times(-2)^4$$
$$+6\times x^1\times(-2)^5$$
$$+1\times x^0\times(-2)^6$$

$$=x^6-12x^5+60x^4-160x^3$$
$$+240x^2-192x+64$$

(b) Expand and simplify $\left(3x+\frac{1}{2}\right)^4$

$$1\times(3x)^4\times\left(\frac{1}{2}\right)^0$$
$$+4\times(3x)^3\times\left(\frac{1}{2}\right)^1$$
$$+6\times(3x)^2\times\left(\frac{1}{2}\right)^2$$
$$+4\times(3x)^1\times\left(\frac{1}{2}\right)^3$$
$$+1\times(3x)^0\times\left(\frac{1}{2}\right)^4$$

$$=81x^4+54x^3+\frac{27}{2}x^2+\frac{3}{2}x+\frac{1}{16}$$

(c) Expand and simplify $(x^2+5)^3$

$$1\times(x^2)^3\times5^0$$
$$+3\times(x^2)^2\times5^1$$
$$+3\times(x^2)^1\times5^2$$
$$+1\times(x^2)^0\times5^3$$

$$=x^6+15x^4+75x^2+125$$

(d) Expand and simplify $\left(\frac{x}{3}-1\right)^5$

$$1\times\left(\frac{x}{3}\right)^5\times(-1)^0+5\times\left(\frac{x}{3}\right)^4\times(-1)^1$$
$$+10\times\left(\frac{x}{3}\right)^3\times(-1)^2+10\times\left(\frac{x}{3}\right)^2\times(-1)^3$$
$$+5\times\left(\frac{x}{3}\right)^1\times(-1)^4+1\times\left(\frac{x}{3}\right)^0\times(-1)^5$$

$$=\frac{1}{243}x^5-\frac{5}{81}x^4+\frac{10}{27}x^3-\frac{10}{9}x^2$$
$$+\frac{5}{3}x-1$$

(e) Expand and simplify $\left(\frac{3}{2}-5y\right)^4$

$$1\times\left(\frac{3}{2}\right)^4\times(-5y)^0$$
$$+4\times\left(\frac{3}{2}\right)^3\times(-5y)^1$$
$$+6\times\left(\frac{3}{2}\right)^2\times(-5y)^2$$
$$+4\times\left(\frac{3}{2}\right)^1\times(-5y)^3$$
$$+1\times\left(\frac{3}{2}\right)^0\times(-5y)^4$$

$$=\frac{81}{16}-\frac{135}{2}y+\frac{675}{2}y^2-750y^3+625y^4$$

(f) Expand and simplify $\left(2x-\sqrt{3}\right)^3$

$$1\times(2x)^3\times\left(-\sqrt{3}\right)^0$$
$$+3\times(2x)^2\times\left(-\sqrt{3}\right)^1$$
$$+3\times(2x)^1\times\left(-\sqrt{3}\right)^2$$
$$+1\times(2x)^0\times\left(-\sqrt{3}\right)^3$$

$$=8x^3-12\sqrt{3}\,x^2+18x-6\sqrt{3}$$