Maclaurin Series

Find the terms up to and including x^4 of the Maclaurin series for each of these functions.

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^n(0)}{n!}x^n$$

(a)

$$f(x) = \cos(2x)$$

$f(x)$	$\cos(2x)$	$f(0)$	1
$f'(x)$	$-2\sin(2x)$	$f'(0)$	0
$f''(x)$	$-4\cos(2x)$	$f''(0)$	-4
$f'''(x)$	$8\sin(2x)$	$f'''(0)$	0
$f^4(x)$	$16\cos(2x)$	$f^4(0)$	16

$$\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 + ..$$

(b)

$$f(x) = e^{-5x}$$

$f(x)$	e^{-5x}	$f(0)$	1
$f'(x)$	$-5e^{-5x}$	$f'(0)$	-5
$f''(x)$	$25e^{-5x}$	$f''(0)$	25
$f'''(x)$	$-125e^{-5x}$	$f'''(0)$	-125
$f^4(x)$	$625e^{-5x}$	$f^4(0)$	625

$$e^{-5x} = 1 - 5x + \frac{25}{2}x^2 - \frac{125}{6}x^3 + \frac{625}{24}x^4 - ..$$

(c)

$$f(x) = \sin\left(-\frac{1}{2}x\right)$$

$f(x)$	$\sin\left(-\frac{1}{2}x\right)$	$f(0)$	0
$f'(x)$	$-\frac{1}{2}\cos\left(-\frac{1}{2}x\right)$	$f'(0)$	$-\frac{1}{2}$
$f''(x)$	$-\frac{1}{4}\sin\left(-\frac{1}{2}x\right)$	$f''(0)$	0
$f'''(x)$	$\frac{1}{8}\cos\left(-\frac{1}{2}x\right)$	$f'''(0)$	$\frac{1}{8}$
$f^4(x)$	$\frac{1}{16}\sin\left(-\frac{1}{2}x\right)$	$f^4(0)$	0

$$\sin\left(-\frac{1}{2}x\right) = -\frac{1}{2}x + \frac{1}{48}x^3 - ..$$

(d)

$$f(x) = \ln(1 + 4x)$$

$f(x)$	$\ln(1+4x)$	$f(0)$	0
$f'(x)$	$\frac{4}{1+4x}$	$f'(0)$	4
$f''(x)$	$-\frac{16}{(1+4x)^2}$	$f''(0)$	-16
$f'''(x)$	$\frac{128}{(1+4x)^3}$	$f'''(0)$	128
$f^4(x)$	$-\frac{1536}{(1+4x)^4}$	$f^4(0)$	-1536

$$\ln(1+4x) = 4x - 8x^2 + \frac{64}{3}x^3 - 64x^4$$