Applied Differentiation Problems

(a) A rectangle has a width x cm and a length $(30 - 2x)$ cm. Using calculus, find the maximum area of the rectangle.

(b) A car sales company sells x cars per week. Its revenue R per week is given by the equation $R = 0.2x^2 - 10x + 1750$. Using differentiation, find the number of cars which generates the maximum revenue, and the value of this revenue.

(a) $A = 30x - 2x^2$

$\frac{dA}{dx} = 30 - 4x$

When $\frac{dA}{dx} = 0 \quad x = 7.5\text{cm}$

$A = 112.5\text{cm}^2$

(b) $\frac{dR}{dx} = 0.4x - 10$

$x = 25$

$R = £1625$

(a) The cost C of a car journey when driving at a speed of x mph is given by $C = \frac{720}{x} + 0.2x + 6$. Using differentiation, find the value of x that minimises the cost, and the minimum value of C.

(b) The volume of a box is given by $V = x(5 - x)^2$. Use calculus to find the maximum volume of the box, and the value of x for which this occurs.

(a) $\frac{dC}{dx} = -\frac{720}{x^2} + 0.2$

$\frac{720}{x^2} = 0.2 \Rightarrow x = 60$

$C = 30$

(b) $V = 25x - 10x^2 + x^3$

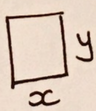
$\frac{dV}{dx} = 25 - 20x + 3x^2$

$x = 5$ or $x = 5/3$
not valid

$A = \frac{500}{27}$

(a) A picture frame has a perimeter of 120 cm. If the width of the frame is x cm, then show that the height of the frame is $(60 - x)$ cm. Hence use calculus to find the value of x that gives a maximum area for the frame. Calculate this maximum area.

(b) A farmer has enough stone for 80 m of dry stone walling. He wants to create a field with the largest area possible. Find the dimensions of the field that gives this maximum area.

(a) 

$2x + 2y = 120$

$2y = 120 - 2x$

$y = 60 - x$

$A = x(60 - x)$

$A = 60x - x^2$

$\frac{dA}{dx} = 60 - 2x \Rightarrow x = 30$

$A = 900\text{cm}^2$.

(b) $A = x(40 - x)$

$A = 40x - x^2$

$\frac{dA}{dx} = 40 - 2x \Rightarrow x = 20$

$A = 400\text{m}^2$