Algebraic Proof with Odds and Evens

Write down algebraic expressions for:

(a) two different odd numbers

(b) two consecutive numbers

(c) two consecutive even numbers

(d) two different odd numbers squared

(a) Prove algebraically that the sum of any two odd numbers is always even.

(b) Prove algebraically that the product of an odd number and an even number is always even

(c) Prove algebraically that the sum of two consecutive numbers is always odd

(d) Prove algebraically that the product of two consecutive even numbers is always even

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(a) Prove algebraically that the mean of two consecutive odd numbers is always even

(b) Prove algebraically that the difference between an odd number and an even number is always odd

(c) Prove algebraically that the mean of three consecutive odd numbers is always equal to the middle number

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(a) Prove algebraically that the sum of the squares of two even numbers is always a multiple of 4

(b) Prove algebraically that the difference between the squares of two odd numbers is always a multiple of 4

(c) Prove algebraically that the sum of the squares of two consecutive numbers is always odd

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(a) Show for all integers values of n that $(n + 3)^2 - n(n - 6) + 2$ is always odd

(b) Show for all integer values of n that $(3n + 5)^2 + (3 - n)^2$ is always even

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For example:

(a) $2n+1$ and $2m+1$

(b) n and $n+1$

(c) $2n$ and $2n+2$

(d) $(2n+1)^2$ and $(2m+1)^2$

(a) $2n+1 + 2m+1 = 2n+2m+2$
$= 2(n+m+1)$

(b) $2n(2m+1) = 4nm + 2n$
$= 2(2nm+n)$

(c) $n+n+1 = 2n+1 = 2(n)+1$

(d) $2n(2n+2) = 4n^2 + 4n$
$= 2(2n^2 + 2n)$

(a) $\dfrac{(2n+1)+(2n+3)}{2} = \dfrac{4n+4}{2} = 2n+2$
$= 2(n+1)$

(b) $(2n+1) - 2m = 2n+1-2m$
$= 2(n-m)+1$

(c) $\dfrac{(2n+1)+(2n+3)+(2n+5)}{3} = \dfrac{6n+9}{3}$
$= 2n+3$

(a) $(2n)^2 + (2m)^2 = 4n^2 + 4m^2$
$= 4(n^2 + m^2)$

(b) $(2n+1)^2 - (2m+1)^2 =$
$4n^2 + 4n + 1 - 4m^2 - 4m - 1$
$= 4(n^2 + n - m^2 - m)$

(c) $n^2 + (n+1)^2 = n^2 + n^2 + 2n + 1$
$= 2n^2 + 2n + 1 = 2(n^2 + n) + 1$

(a) $n^2 + 6n + 9 - n^2 + 6n + 2 =$
$12n + 11 = 2(n+6) - 1$

(b) $9n^2 + 30n + 25 + 9 - 6n + n^2$
$= 10n^2 + 24n + 34$
$= 2(5n^2 + 12n + 17)$