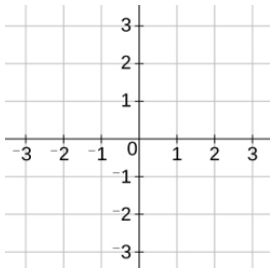
Enlargements Using Matrices

(a)

By considering the unit square, determine the matrix which describes an enlargement about the origin with scale factor 3.



$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

(b)

Describe fully the single transformation represented by the matrix $\begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$

Enlargement with scale factor 2.5 about the origin

(c)

Use matrix algebra to show that an enlargement of scale factor 2 about $(0, 0)$, followed by an enlargement of scale factor 1.5 about $(0, 0)$ is equivalent to an enlargement of scale factor 3 about $(0, 0)$.

$$\begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

(d)

The point $(-5, 3)$ is mapped onto the point (a, b) when enlarged by a scale factor 2 about the origin. Using matrix algebra, find the values of a and b.

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -5 \\ 3 \end{pmatrix} = \begin{pmatrix} -10 \\ 6 \end{pmatrix}$$

$$a = -10, b = 6$$

(e)

The unit square OABC with coordinates O(0, 0), A(0, 1), B(1, 1) and C(1, 0) is mapped to OA'B'C' under matrix $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$. Use matrix algebra to find the coordinates of A', B' and C'.

$$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \quad \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$$

(f)

The point (c, d) is mapped onto the point $(-1, -4)$ when enlarged by a scale factor 0.5 about the origin. Using matrix algebra, find the values of c and d.

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

$$c = -2, d = -8$$

(g)

Use matrix algebra to show that an enlargement of scale factor 2 about $(0, 0)$, followed by an enlargement of scale factor -0.5 about $(0, 0)$ is the same as a rotation of $180°$ about the origin.

$$\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(h)

The point $(a, 3)$ is mapped to the point $(6, 2a)$ when enlarged with scale factor b about the origin. Use matrix algebra to find the possible values of a and b.

$$\begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} a \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2a \end{pmatrix}$$

$$a = 3, b = 2 \text{ or } a = -3, b = -2$$

(i)

The point $(x - 4, y)$ is mapped to the point $(2y, 2x - 18.5)$ when transformed under the matrix $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$. Find the values of x and y.

$$x = 3, y = 2.5$$