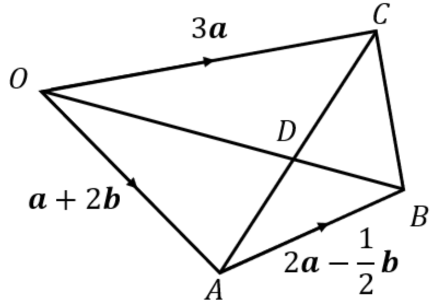
Vector Proof – Equating Coefficients

(a)

$OABC$ is a quadrilateral, where $\overrightarrow{OC} = 3\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} + 2\boldsymbol{b}$ and $\overrightarrow{AB} = 2\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$. The point D is on OB and AC such that $OD : OB = \lambda : 1$ and $AD : AC = \mu : 1$. By finding two ways to express the vector $\overrightarrow{OD}$, find the values of λ and μ.



$$\overrightarrow{OD} = \lambda\overrightarrow{OB} = \lambda\left(3\boldsymbol{a} + \frac{3}{2}\boldsymbol{b}\right)$$
$$\overrightarrow{OD} = \overrightarrow{OA} + \mu\overrightarrow{AC}$$
$$\overrightarrow{OD} = \boldsymbol{a} + 2\boldsymbol{b} + \mu(2\boldsymbol{a} - 2\boldsymbol{b})$$

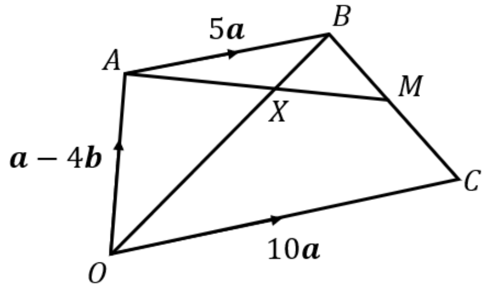
Coefficients of $\boldsymbol{a}$:
$$3\lambda = 1 + 2\mu$$

Coefficients of $\boldsymbol{b}$:
$$\frac{3}{2}\lambda = 2 - 2\mu$$

$$\lambda = \frac{2}{3}, \mu = \frac{1}{2}$$

(b)

$OABC$ is a trapezium, where $\overrightarrow{OC} = 10\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} - 4\boldsymbol{b}$ and $\overrightarrow{AB} = 5\boldsymbol{a}$. M is the midpoint of the line BC. The point X is on OB and AM such that $OX : OB = \lambda : 1$ and $AX : AM = \mu : 1$. Find the values of λ and μ and the vector $\overrightarrow{OX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



$$\overrightarrow{OX} = \lambda\overrightarrow{OB} = \lambda(6\boldsymbol{a} - 4\boldsymbol{b})$$
$$\overrightarrow{OX} = \overrightarrow{OA} + \mu\overrightarrow{AM}$$
$$\overrightarrow{OX} = \boldsymbol{a} - 4\boldsymbol{b} + \mu(7\boldsymbol{a} + 2\boldsymbol{b})$$

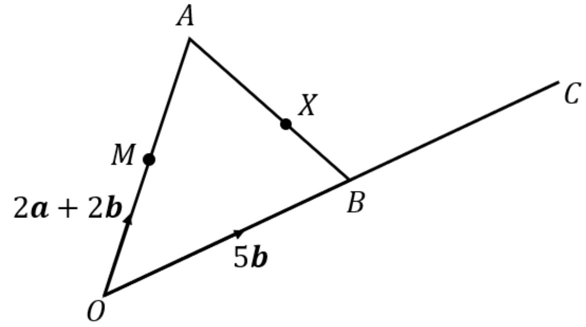
Coefficients of $\boldsymbol{a}$:
$$6\lambda = 1 + 7\mu$$

Coefficients of $\boldsymbol{b}$:
$$-4\lambda = -4 + 2\mu$$

$$\lambda = \frac{3}{4}, \mu = \frac{1}{2}, \overrightarrow{OX} = \frac{9}{2}\boldsymbol{a} - 3\boldsymbol{b}$$

(c)

In the triangle OAB, $\overrightarrow{OB} = 5\boldsymbol{b}$ and $\overrightarrow{OM} = 2\boldsymbol{a} + 2\boldsymbol{b}$, where M is the midpoint of OB. OC is the line OB produced and $\overrightarrow{OB} = \overrightarrow{BC}$. The point X is on the line AB such that $AX : AB = \lambda : 1$. Given that MXC is a straight line, find the value of λ and the vector $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



$$\overrightarrow{MX} = \mu\overrightarrow{MC} = \mu(-2\boldsymbol{a} + 8\boldsymbol{b})$$
$$\overrightarrow{MX} = \overrightarrow{MA} + \lambda\overrightarrow{AB}$$
$$\overrightarrow{MX} = 2\boldsymbol{a} + 2\boldsymbol{b} + \lambda(-4\boldsymbol{a} + \boldsymbol{b})$$

Coefficients of $\boldsymbol{a}$:
$$-2\mu = 2 - 4\lambda$$

Coefficients of $\boldsymbol{b}$:
$$8\mu = 2 + \lambda$$

$$\lambda = \frac{2}{3}, \mu = \frac{1}{3}, \overrightarrow{MX} = -\frac{2}{3}\boldsymbol{a} + \frac{8}{3}\boldsymbol{b}$$