Numerical Methods Revision

(a)

$$x_{n+1} = \frac{e^{x_n}}{x_n + 3}$$

Using $x_1 = 1$ find the values of x_2, x_3 and x_4 to 4 decimal places.

$x_2 = 0.6796, x_3 = 0.5362, x_4 = 0.4834$

(b)

$$f(x) = \ln(2x) - \frac{3}{x}$$

The equation $f(x) = 0$ has a single root α.

Show that α lies in the interval $[2, 2.1]$.

$f(2) = -0.114, f(2.1) = 0.007$

Since there is a change of sign and the function is continuous, there is a root

(c)

$$f(x) = x^3 - x - 2\sin x$$

Show that the equation $f(x) = 0$ can be written as the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2\sin x_n}{x_n}}$$

(d)

Show that the equation $x^3 - \sqrt{x} = 3$ has the solution $x = 1.623$ to 3 decimal places.

$f(x) = x^3 - \sqrt{x} - 3$

$f(1.6225) = -0.0025$

$f(1.6235) = 0.0050$

Since there is a change of sign, a root lies in the interval $[1.6225, 1.6235]$

(e)

$$x_{n+1} = 2 + 3\ln x_n$$

Using $x_0 = 6$, find x_1, x_2 and x_3. Hence find the root of the equation $2 + 3\ln x - x = 0$ to 3 decimal places.

$x_1 = 7.375, x_2 = 7.994, x_3 = 8.236$

$x = 8.376$

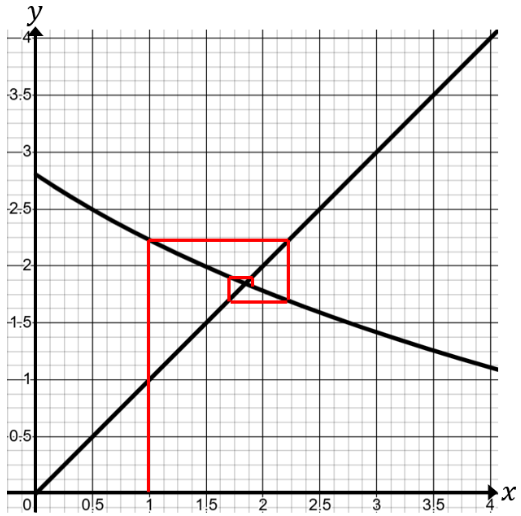
(f)

The iterative formula

$$x_{n+1} = 5 + 2\ln\left(\frac{1}{x_n + 3}\right)$$

is used to find an approximate solution to the equation $5 - x + 2\ln\left(\frac{1}{x+3}\right) = 0$

Given $x_1 = 1$, show using a cobweb or staircase diagram how the convergence towards the root takes place.



(g)

Use the Newton-Raphson method with $x_1 = 1$ to find an approximate root to the equation $5 + x - e^x = 0$ to 3 decimal places.

$$x_{n+1} = x_n - \frac{5 + x_n - e^{x_n}}{1 - e^{x_n}}$$

$x_2 = 1.839, x_3 = 1.943, x_4 = 1.937$

(h)

The height of a ball as a function of time can be modelled by the equation

$$h(t) = 5\sqrt{t} - \frac{1}{2}t^3 \qquad t \geq 0$$

Taking a first approximation of $t = 2$ s, use the Newton-Raphson method once to find a second approximation for the time when the ball hits the ground. Give your answer to 3 significant figures.

$t = 2.73$ s