

Probability from Independent Events

(a) Arthur tosses a fair coin three times. Find the probability that the coin lands on tails exactly once.

$$(a) THH + HTH + HHT \\ = 3 \times \frac{1}{8} = \frac{3}{8}$$

(b) Iris and Usma play three games of tennis. The probability that Iris wins a game of tennis is $\frac{3}{8}$. Find the probability that Usma wins at least one game of tennis.

$$(b) 1 - \left(\frac{5}{8}\right)^3 = \frac{387}{512}$$

(a) Erik throws a biased coin twice. The probability of the coin landing on tails twice is $\frac{25}{81}$. Find the probability that the coin lands on heads twice.

$$(a) P(T) = \frac{5}{9} \quad \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

(b) A spinner has two unequal sections coloured black and white. When the spinner is spun three times, the probability that it lands on black three times is 0.064. Find the probability that when the spinner is spun three times, it lands on white exactly twice.

$$(b) \sqrt[3]{0.064} = \frac{2}{5} \quad \begin{array}{l} WWB \\ WBW \\ BWW \end{array} \\ 3 \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{54}{125}$$

(a) There are 20 counters in a bag - some are red and the rest are blue. A counter is chosen at random from the bag, it is replaced and then a second counter is chosen. The probability that two red counters are chosen is 0.09. Find the number of red counters, the number of blue counters and the probability of choosing one counter of each colour.

$$(a) P(RR) = \frac{9}{100} \quad P(R) = \frac{3}{10} \\ 6 \text{ red, } 14 \text{ blue} \quad \frac{3}{10} \times \frac{7}{10} \times 2 \\ = \frac{21}{50}$$

(b) Peta and Paul play one game of tennis and one game of pool against each other. The probability that Peta wins the game of pool is 0.66. The probability that Paul wins both games is 0.272. Find the probability that they win exactly one game each.

$$(b) P(\text{Paul, Pool}) = 0.34 \\ P(\text{Paul, Tennis}) = \frac{0.272}{0.34} = 0.8 \\ P(\text{one each}) = 0.34 \times 0.2 + 0.66 \times 0.8 \\ = \frac{149}{250} = 0.596$$

A spinner has three unequal sections - white, grey and black. The grey section is twice the size of the black section. When the spinner is spun four times, the probability of it landing on white four times in a row is 0.2401. Find the probability of the spinner landing on grey four times in a row.

$$\sqrt[4]{0.2401} = \frac{7}{10} = P(W) \\ P(G) = \frac{2}{10} \\ P(GGGG) = \left(\frac{2}{10}\right)^4 = \frac{1}{625}$$